Matlab/Freemat/Octave/Scilab: Solution of Linear Systems of Equations

A linear system[1] of equations can be written as a matrix-vector equation. Often a linear system of equations has the same number of equations as it has *unknowns*. In this case the matrix will be square and this is the case that is considered in this document.

A linear system of equations can be written in the matrix-vector form

$$A\underline{x} = \underline{b},$$

where A is a square matrix, $\underline{b}$ is a given vector and $\underline{x}$ is the vector of *unknowns*.

The solution can be obtained by inverting A, and this method will be considered in this document. However, there is also a direct method of *division* in Matlab/Freemat/Octave/Scilab, which is also more computationally efficient than the former method.

Solution by inverting A

By writing the equation in the form

$$\underline{x} = A^{-1}\underline{b},$$

the solution of the linear system of equations can be found by first inverting A and multiplying the result by $\underline{b}$.

For example the equation

$$\begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 7 \\ 12 \end{pmatrix}$$

may be solved as follows.

```
--> A=[2 1; 3 2]
A =
2 1
3 2
--> b=[7; 12]
b =
7
12
--> inv(A)*b
ans =
2
3
```

[1] Linear Systems and 2x2 Matrices

Solution by division

The recommended method is to use the *division* operator (\) to solve linear systems of equations. For example the system above can be solved as follows.

```
--> A=[2 1; 3 2]
A =
2 1
3 2
--> b=[7; 12]
b =
7
12
--> A\b
ans =
2.0000
3.0000
```

Given the example of a 3×3 system

$$\begin{pmatrix} 1 & 0 & 1 \\ 1 & 2 & 1 \\ 0 & 2 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix},$$

the solution $\begin{pmatrix} -1 \\ 0.5 \\ 2 \end{pmatrix}$ can be found in Matlab/Freemat as follows.

```
A =
1 0 1
1 2 1
 0 2 1
--> b=[1;2;3]
b =
1
2
3
--> A\b
ans =
-1.0000
0.5000
2.0000
```